

MARCO RAMPAZZO

CURRICULUM VITAE, 21 OCTOBER 2021.

PERSONAL INFORMATION

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ACADEMIC ACTIVITY

Current position Assegno di Ricerca, University of Bologna	2021 – now
Previous position PhD student in mathematics, University of Stavanger Supervisor: Michał Kapustka Thesis: “Equivalences of Calabi–Yau manifolds and roofs of projective bundles”	2016 – 2020
Guest positions / Thematic programs Guest of the Paul Sabatier University, Toulouse Funding: Norwegian Research Council mobility grant Host: Laurent Manivel	Spring 2019

EDUCATION

Master’s degree in Physics University of Milan	2016
Bachelor’s degree in Physics University of Milan	2013

RESEARCH INTERESTS

Algebraic varieties: Calabi–Yau varieties, homogeneous varieties and homogeneous vector bundles, Fano varieties with multiple projective bundle structures

Derived categories of coherent sheaves: semiorthogonal decompositions, mutations of exceptional collections, derived equivalence, Fourier–Mukai transform

Birational geometry: roofs of projective bundles, K-equivalence, DK-conjecture

Gauged linear sigma models: multiple geometric phases, phase transitions, variation of GIT

TEACHING

Linear Algebra, exercise classes	fall 2021
Discrete Mathematics, exercise classes	fall 2020
Linear Algebra, exercise classes	fall 2020
Probability and Statistics, exercise classes	spring 2020
Linear algebra, teaching and exercise classes	fall 2019
Linear algebra, exercise classes	fall 2018
Linear algebra, exercise classes	fall 2017

CONFERENCE TALKS

Workshop “Algebraic Geometry days”. <i>Mukai roofs and K3 surfaces</i>	Stavanger, 25–26 November 2019
Conference “Nasjonalt Algebramøte 2019”. <i>Derived equivalence of Mukai roofs: the case of K3 surfaces of degree 12</i>	Oslo, 7–8 November 2019
Seminar of Algebra of the Jagellonian University. <i>Computing Hodge numbers of Calabi–Yau varieties in Grassmannians</i>	Kraków, 11 April 2019
Conference “Nasjonalt Matematikermøte 2018”. <i>A GLSM description for a pair of non birational Calabi–Yau threefolds</i>	Bergen, 12 September 2018
Workshop “Motives of Calabi–Yau manifolds”. <i>A gauged linear sigma model description for a pair of non birational Calabi–Yau threefolds</i>	Kraków, 19–21 May 2018

SEMINARS ORGANIZED

Seminar: <i>Bridgeland stability conditions</i> Organizer together with Simone Billi, Francesco Denisi, Franco Giovenzana, Annalisa Grossi and Mihai–Cosmin Pavel	Bologna – Chemnitz – Nancy, fall 2021.
Seminar: <i>The mathematics of gauged linear sigma models</i> Organizer and speaker	Toulouse, spring 2019.

PUBLICATIONS AND PREPRINTS

1. *PhD Thesis*: Marco Rampazzo. *Equivalences between Calabi–Yau manifolds and roofs of projective bundles*. (2021). <https://doi.org/10.31265/usps.78>
Available online at <https://ebooks.uis.no/index.php/USPS/catalog/book/78>
2. *Publication*: Michał Kapustka, Marco Rampazzo. *Mukai duality and roofs of projective bundles*. (2021). Accepted by the Bulletin of the London Mathematical Society.
3. *Publication*: Michał Kapustka, Marco Rampazzo. *Torelli problem for Calabi–Yau threefolds with GLSM description*. Communications in Number Theory and Physics, Volume 13, No. 4 (2019).
4. *Preprint*: Enrico Fatighenti, Michał Kapustka, Giovanni Mongardi, Marco Rampazzo. *The generalized roof $F(1, 2, n)$: Hodge structures and derived categories*. (2021). Available on arXiv.
5. *Preprint*: Marco Rampazzo. *Calabi–Yau fibrations, simple K -equivalence and mutations*. (2020). e-print: arXiv:2006.06330

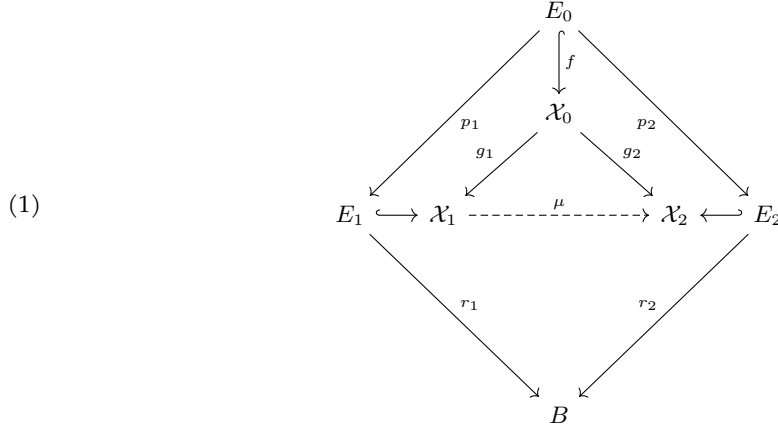
DERIVED CATEGORIES, SEMIORTHOGONAL DECOMPOSITIONS AND ROOFS OF PROJECTIVE BUNDLES

RESEARCH STATEMENT OF MARCO RAMPAZZO

ABSTRACT. It is conjectured that many birational transformations, called K-inequalities, have a categorical counterpart in terms of an embedding of derived categories. In the special case of simple K-equivalence (or more generally K-equivalence), a derived equivalence is expected: I propose a method to prove derived equivalence for a wide class of such cases, based on the proof of derived equivalence of an associated pair of Calabi–Yau varieties. This method is related to the construction of roofs of projective bundles introduced by Kanemitsu. In the same framework, a similar approach applies to prove derived equivalence of pairs of Calabi–Yau fibrations. All these constructions lead to relations in the Grothendieck ring of varieties. Generalizing Kanemitsu’s roof construction leads to a conjectural homological projective duality for zero loci of certain vector bundles on generalized Grassmannians.

DK conjecture and simple K-equivalence. In light of the reconstruction theorem by Bondal and Orlov [BO01], the derived category of coherent sheaves has been established as a powerful invariant in the classification of varieties with ample canonical or anticanonical bundle. However, its role of invariant up to isomorphism breaks down for varieties of trivial canonical class: many counterexamples, in the form of pairs of Calabi–Yau varieties which are derived equivalent but not birationally equivalent, have been found. Nonetheless, the role of derived category as a *birational* invariant beyond Fano and general type varieties is suggested by the DK conjecture of Bondal, Orlov and Kawamata [BO02, Kaw02] at the price of introducing some additional assumption: in fact it is expected that many birational transformations, called K-equivalences and K-inequalities, imply equivalences or embeddings of derived categories. Among K-equivalences, a special place is occupied by *simple K-equivalences*: two varieties \mathcal{X}_1 and \mathcal{X}_2 are said to be simply K-equivalent if there exists a birational morphism $\mathcal{X}_1 \dashrightarrow \mathcal{X}_2$ resolved by two smooth blowups $\mathcal{X}_0 \rightarrow \mathcal{X}_i$ with isomorphic exceptional loci $E_0 \subset Z$. Kanemitsu proved that in each of such cases E_0 is isomorphic to a variety with two different projective bundles structure of the same rank [Kan18, Theorem 0.2]. More precisely, the following diagram exists for

any simple K-equivalence μ :



In Diagram 1, for $i \in \{1; 2\}$, E_i are the centers of the blowups, r_i are smooth extremal contractions and p_i are projective bundles of rank $r = \text{Cod}_{\mathcal{X}_i} E_i$. Moreover, the outer square of Diagram 1 is commutative, therefore let us call $\pi := r_i \circ p_i$. Fibers of π are very special Fano varieties called *roofs of projective bundles* (or simply *roofs*), i.e. Fano manifolds X of Picard number two which admit two projective bundle structure of the same rank $h_1 : X \simeq \mathbb{P}\mathcal{E}_1 \rightarrow X_1$ and $h_2 : X \simeq \mathbb{P}\mathcal{E}_2 \rightarrow X_2$, such that (X_i, \mathcal{E}_i) is a Mukai pair and there is a line bundle L which restricts to $\mathcal{O}(1)$ on the fibers of both h_1 and h_2 . Kanemitsu gave a partial classification of such objects, which is completely exhaustive in the case where X is a quotient of a simply connected, semisimple Lie group by a parabolic subgroup [Kan18, Theorem 0.3]. This class of K-equivalences includes, for example, all standard flops, Grassmannian flops and Mukai flops of smooth projective varieties.

Roofs are interesting objects in their own right: in fact, one can associate a pair of Calabi–Yau varieties to the data of a general hyperplane section on a roof [KR20, Definition 2.5], [R20, Lemma 2.4]. In this class lie several of the known examples of non birational derived equivalent Calabi–Yau pairs [Muk98, Kuz18, KR17, KR20, R20]. In a joint work by Michał Kapustka and myself, we show the existence of an isomorphism of Hodge structures mapping the middle cohomology of each of the Calabi–Yau varieties of the pair to a part of the middle cohomology of the hyperplane section of the roof which defines them [KR20, Section 3]. In particular, this proves that a pair of odd-dimensional Calabi–Yau varieties arising from such construction is Hodge equivalent. Moreover, this isomorphism allows us to prove that a pair of K3 surfaces defined by a general hyperplane section of a roof is non isomorphic [KR20, Section 4.4], and derived equivalent in the case of degree 12 K3 surfaces defined as zero loci of twisted Ottaviani bundles in five dimensional quadrics [KR20, Proposition 4.7].

In higher dimension, an effective approach to prove derived equivalence for Calabi–Yau pairs arising by a roof is by means of mutations of exceptional collections. More precisely, let us call $M \subset X$ the zero locus of a general hyperplane section $\sigma \in H^0(X, L)$ where X is a roof of \mathbb{P}^{r-1} -bundles, and let (Y_1, Y_2) be a pair of Calabi–Yau varieties where $Y_i = Z(h_{i*}\sigma)$. One has the

following two semiorthogonal decomposition for $i \in \{1, 2\}$:

$$(2) \quad D^b(M) = \langle k_{i*} h_i^*|_{h_i^{-1}(Y_i)} D^b(Y_i), j^* h_i^* D^b(X_i) \otimes L, \dots, j^* h_i^* D^b(X_i) \otimes L^{\otimes r} \rangle$$

where $k_i : h_i^{-1}(Y_i) \rightarrow M$ and $j : M \hookrightarrow X$. In several cases [Kuz18, KR17, R20] it has been proved that Y_1 and Y_2 are derived equivalent by showing that they have the same semiorthogonal complement. This can be achieved by mutating full exceptional collections of such semiorthogonal complements until they can be expressed as the same list of exceptional objects. In fact, the presence of collections like the ones of Equation 2 and the examples above motivate the following conjecture:

Conjecture 1. [KR20, Conjecture 2.6] *Let X be a roof of projective bundles and $\sigma \in H^0(X, L)$ a general hyperplane section. Then Y_1 and Y_2 are derived equivalent, where $Y_i = Z(h_{i*}\sigma)$.*

In all the worked examples the Calabi–Yau pairs are \mathbb{L} -equivalent, i.e. they satisfy the following equation in the Grothendieck ring of varieties:

$$(3) \quad ([Y_1] - [Y_2])\mathbb{L}^{r-1} = 0$$

where r is the rank of the projective bundle structures of the associated roof. Therefore, in light of the evidences, we

Conjecture 2. [KR20, Section 2.1]. *Let X be a roof of projective bundles and $\sigma \in H^0(X, L)$ a general hyperplane section. Then Y_1 and Y_2 are \mathbb{L} -equivalent in the Grothendieck ring of varieties, i.e. they satisfy Equation 3.*

It is an interesting problem to prove the conjectures above with general arguments and shed some light on the still open problem of understanding the relation between derived equivalence and \mathbb{L} -equivalence of Calabi–Yau varieties.

This approach based on mutations can be used to derive a more general result. In fact, by Diagram 1 and Orlov’s blowup formula [Orl93, Theorem 4.3] one can write the following semiorthogonal decompositions for $i \in \{1, 2\}$:

$$(4) \quad D^b(\mathcal{X}_0) = \langle f_* p_i^* D^b(E_i) \mathcal{L}^{\otimes(-r)}, \dots, f_* p_i^* D^b(E_i) \otimes \mathcal{L}^{\otimes(-1)}, g_i^* D^b(X_i) \rangle$$

where \mathcal{L} is the Grothendieck line bundle of both the projective bundle structures of E_0 . In this context, I prove that if there exists a sequence of mutations for the collections of Equation 2 which gives a derived equivalence $D^b(Y_1) \simeq D^b(Y_2)$, then \mathcal{X}_1 and \mathcal{X}_2 are derived equivalent as well, under mild cohomological hypotheses which are satisfied by all the known examples [R20, Proposition 5.2]. This approach immediately applies to many cases of simple K-equivalence, for example Mukai flops and standard flops, provided that the extremal contractions r_i of Diagram 1 are locally trivial fibrations [R20, Theorem 5.3]. Motivated by this evidence, I expect that for every simply K-equivalent map $\mu : \mathcal{X}_1 \dashrightarrow \mathcal{X}_2$ with homogeneous exceptional locus such that the maps r_i are locally trivial, there exists an equivalence of categories $D^b(\mathcal{X}_1) \simeq D^b(\mathcal{X}_2)$.

Derived equivalence of Calabi–Yau fibrations. In a work by Bridgeland and Maciocia [BM02], pairs of fibrations of K3 surfaces and elliptic curves have been proved to be derived equivalent by extending a fiberwise Fourier–Mukai duality to the whole fibration. I consider higher dimensional examples of such constructions: more precisely, let $\pi : E_0 \rightarrow B$ be a locally trivial family of roofs with fiber $\pi^{-1}(b) \simeq X$ for every $b \in B$ and let \mathcal{L} be the Grothendieck line bundle of both the projective bundle structures $p_i : E_0 \rightarrow E_i$. Then, given a general section

$\Sigma \in H^0(E_0, \mathcal{L})$ the zero loci of $p_{i*}\Sigma$ are Calabi–Yau fibrations over B , and the fibers over a general $b \in B$ are a Calabi–Yau pair related to the roof X , i.e. the pushforwards of a general hyperplane section of X to the bases of the two projective bundle structures [R20, Lemma 2.11].

In this setting, in order to prove derived equivalence, the approach is based again on mutations of exceptional objects on the fiber: in fact, calling Z_1 and Z_2 the Calabi–Yau fibrations, there exist the following semiorthogonal decompositions:

$$(5) \quad D^b(N) = \langle D^b(E_i), \dots, D^b(E_i) \otimes \mathcal{L}^{\otimes(r-2)}, D^b(Z_i) \rangle$$

Once again, one can observe a striking similarity with Equation 2. In fact, Z_1 and Z_2 are derived equivalent if Y_1 and Y_2 can be proven to be derived equivalent by identifying their semiorthogonal complements in Equation 2 with a sequence of mutations [R20, Theorem 4.5]. As in the case of simple K-equivalence, this approach immediately gives derived equivalence for several cases [R20, Corollary 4.12 - 4.13]: one particularly interesting example is given by a pair of Calabi–Yau eightfolds fibered over \mathbb{P}^5 in Calabi–Yau threefolds. The fibers over a general point $b \in \mathbb{P}^5$ are a pair of Calabi–Yau threefolds related to the roof $F(2, 3, 5)$ described in [KR17], which are derived equivalent but not birationally equivalent [R20, Theorem 6.1].

Towards a proof of derived equivalence for all homogeneous roofs. The program of proving derived equivalence of Calabi–Yau pairs related to homogeneous roofs, alongside with the outcome of infinite series of examples of derived equivalent Calabi–Yau varieties which are expected to be not birationally equivalent, is motivated by the previous results on simple K-equivalence and Calabi–Yau fibrations: in fact, the completion of such program would imply that the DK-conjecture holds for simple K-equivalences resolved in a locally trivial fibration, and a derived equivalence for all the associated Calabi–Yau fibrations. In several of the remaining cases, full exceptional collections for the Grassmannians X_1 and X_2 are known: however, the problem of determining the right sequence of mutations is technically very challenging.

Flops of local Calabi–Yau varieties. The geometry of a pair of Calabi–Yau varieties related by a roof allows to construct a flop between the total spaces of two vector bundles. In fact, let us consider a roof $X \simeq \mathbb{P}(\mathcal{E}_1) \simeq \mathbb{P}(\mathcal{E}_2)$, and call \mathcal{L} the Grothendieck line bundle of both the projective bundle structures. Then, the total space $\text{Tot}(\mathcal{L})$ can be interpreted as a blowup of both $\text{Tot}(\mathcal{E}_1)$ and $\text{Tot}(\mathcal{E}_2)$ with center in the related Calabi–Yau manifolds, i.e. the zero loci of pushforwards of a general section of \mathcal{L} . Both $\text{Tot}(\mathcal{E}_1)$ and $\text{Tot}(\mathcal{E}_2)$ are Calabi–Yau, and this construction defines a flop $\text{Tot}(\mathcal{E}_1) \dashrightarrow \text{Tot}(\mathcal{E}_2)$ resolved by $\text{Tot}(\mathcal{L})$. Diagram 1 describes such behavior in families. In light of Conjecture 1 and [R20], it is reasonable to expect that every roof of projective bundles gives rise to a pair of derived equivalent total spaces related by a flop. In fact, the results of [Ued16] and [Mor19] in this direction can be deduced by means of [R20, Theorem 6.7] by the results of [Kuz18] and [KR17] on the associated pairs of Calabi–Yau varieties arising from roof constructions.

Homological projective duality for sections of vector bundles on Grassmannians. Homological projective duality [Kuz07] is a powerful tool to describe derived categories of algebraic varieties, by means of a systematic study of the effect on a Lefschetz semiorthogonal decomposition of the operation of intersecting with linear spaces, and comparing the resulting decomposition with the one of the intersection of a second variety (called the *homologically projectively dual*) with the dual linear space. However, only few cases of homologically projectively dual pairs are

currently known. In the paper [FKMR21] Enrico Fatighenti, Michał Kapustka, Giovanni Mongardi and myself consider the generalization of a roof to the case when the two projective bundle structures are allowed to have different relative dimensions, and the associated pair (Y_1, Y_2) of zero loci of pushforwards of a general section of $\mathcal{O}(1, 1)$ to its two (generalized) Grassmannians. We classify these *generalized homogeneous roofs*, with particular attention to $F(1, 2, n)$. This variety admits a \mathbb{P}^{n-2} -bundle structure h_1 over \mathbb{P}^{n-1} and a \mathbb{P}^1 -bundle structure h_2 over $G(2, n)$, in the associated pair (Y_1, Y_2) the variety Y_2 is a smooth Fano while Y_1 is a collection of points. With methods based on the B -brane categories of critical loci of a suitable gauged linear sigma model, we prove that $D^b Y_1$ embeds into $D^b Y_2$. This suggests that Y_2 admits a full exceptional collection. The preimage under h_2 of a general hyperplane section of $G(2, n)$, for n even, is the symplectic flag $IF(1, 2, n)$, which has itself two projective bundle structures to \mathbb{P}^{n-1} and $G(2, n) \cap H \simeq IG(2, n)$. In this last case some preliminary results (the case $n = 6$) suggest that the associated pair $(Y_1^{(1)}, Y_2^{(1)})$ of zero loci satisfies the same derived embedding statement. More generally, we introduce the following construction: given a generalized homogeneous roof X with two projective bundle structures $h_i : X \rightarrow B_i$, we fix a section $(S, \sigma_1, \dots, \sigma_k)$ of $\mathcal{O}(1, 1) \oplus \mathcal{O}(0, 1)^{\oplus k}$ (this corresponds to the choice of a section of $\mathcal{O}(1, 1)$ and k pullbacks of hyperplane sections from B_2). Then, we define the variety $Y_2^{(k)}$ as the zero locus of the corresponding section of $h_{2*}\mathcal{O}(1, 1) \oplus \mathcal{O}(1)^{\oplus k}$. We also define a variety $Y_1^{(k)}$ as the degeneracy locus of rank smaller or equal than k of the morphism $f_{(S, \sigma_1, \dots, \sigma_k)} : \mathcal{O} \oplus \mathcal{O}(1)^{\oplus k} \rightarrow h_{1*}\mathcal{O}(1, 1)$ defined by the same section $(S, \sigma_1, \dots, \sigma_k)$. In this way, varying k , we construct an inclusion-reversing tower of pairs where $Y_2^{(k)} \subset Y_2^{(k-1)}$ and $Y_1^{(k)} \supset Y_1^{(k-1)}$. Together with Enrico Fatighenti, Michał Kapustka and Giovanni Mongardi we formulate the following:

Conjecture 3. *The homological projective dual of $Y_2 := Z(S)$ is the universal degeneracy locus:*

$$\mathcal{Y}_1 := \{(x, \sigma) \in B_1 \times \mathbb{P}(H^0(B_2, \mathcal{O}(1))) \text{ s.t. } f_{S, \sigma} : \mathcal{O} \oplus \mathcal{O}(1) \rightarrow h_{1*}\mathcal{O}(1, 1) \text{ has rank } < 2\}.$$

In the case $X = F(1, 2, n)$ we plan to construct a full Lefschetz exceptional collection for $Z(S)$ by adapting the results of [LX19] to our setting, and to use such result to prove the conjecture above. To address more general cases, new techniques are most likely needed, it is part of our project to investigate this problem further.

Gauged linear sigma models and matrix factorization categories. Knörrer periodicity [Orl12, Shi12] suggests an intriguing phenomenon: each of the Calabi–Yau varieties of a dual pair arising from a roof construction, in every known example, can be identified with the critical locus of a phase of a gauged linear sigma model whose B -brane category can be described, in terms of Knörrer periodicity, as the derived category of coherent sheaves of the Calabi–Yau. Hence, derived equivalence lifts to an equivalence of matrix factorization categories. It would be interesting to find a way to understand such derived equivalence in terms of an equivalence of B -brane categories on a non Abelian GLSM by an explicit description of the model and the associated phase transition as illustrated in [ADS15] for the Pfaffian-Grassmannian Calabi–Yau pair. This is possible, for instance, in the case discussed in [KR17] and the example of eight-dimensional Calabi–Yau fibrations considered in [R20]. Moreover, a GLSM description for the pair associated to the generalized roof $F(1, 2, n)$ has been found in [FKMR21], together with an embedding of B -brane categories. In this last case the critical loci are a general type variety of dimension zero and a Fano variety of dimension $2n - 6$.

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